ARJUNA

JEE

LECTURE NO. - 3

Basic Math

MSM SIR

ARJUNA

TABLE OF CONTENT

01 RATIONAL INEQUALITIES

02 Questions Practice

KAHAANI AB TAK

'RULE'

1. *Denominator can never be zero .*

For Example : $\ln \frac{1}{x-1}$, $x \neq 1$.

OR,

In other words , *Multiplicative inverse of zero doesn't exists.*

'2' → (multiplicative inverse) → '$\frac{1}{2}$' $\Rightarrow 2 \times \frac{1}{2} = 1$

'3' → '$\frac{1}{3}$' $\Rightarrow 3 \times \frac{1}{3} = 1$

(any number) × (Its multiplicative Inverse) = 1

'0' * (Uske multiplicative Inverse) = 1
(Not possible)

Multiplicative inverse of '0' i.e. $\left(\frac{1}{0}\right)$ D.N.E.

$\frac{1}{0} = \infty$ (assume)

$\Rightarrow 0.\infty = 1$

- $0.\infty + 0.\infty + 0.\infty = 3$

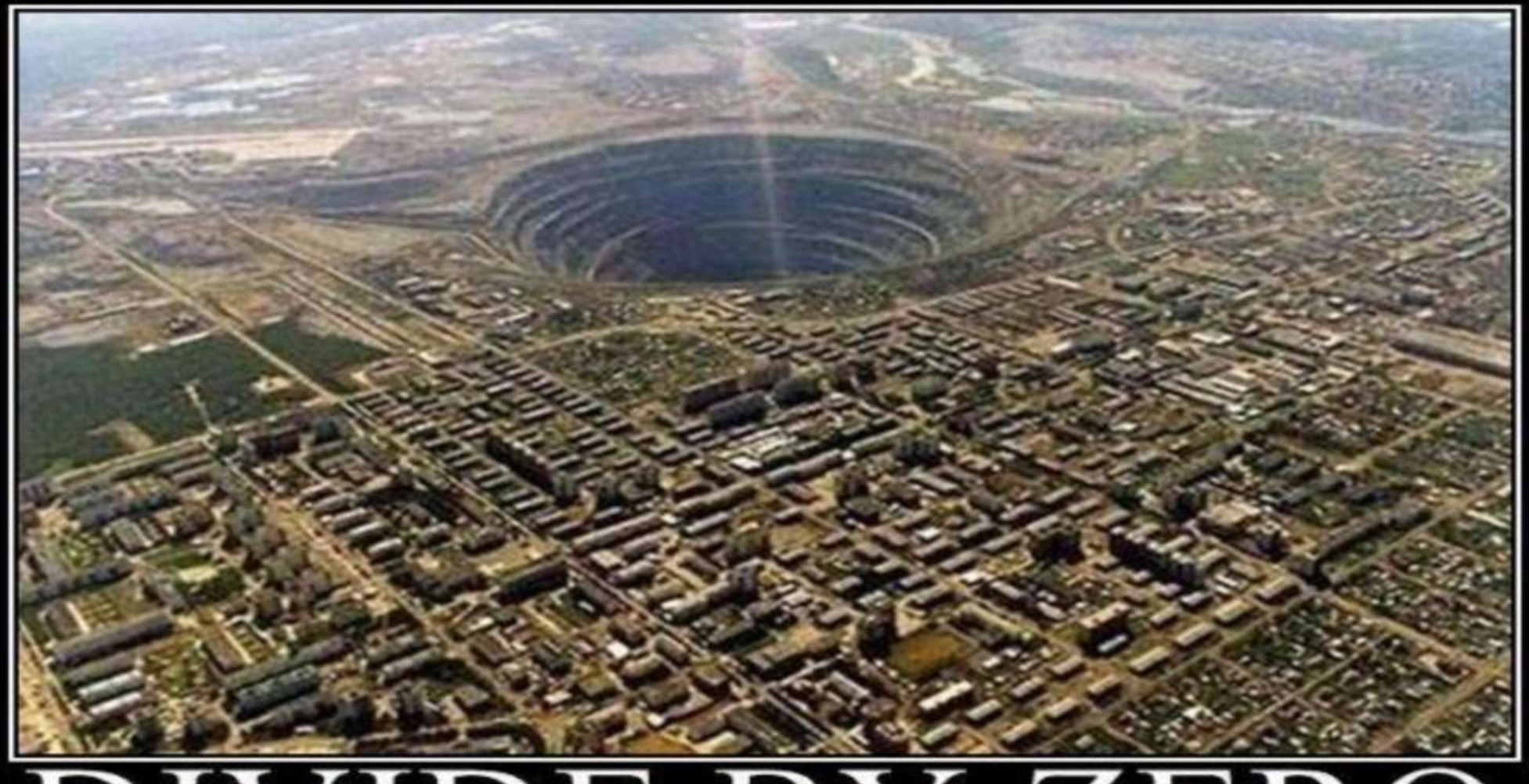
LHS

$(0+0+0)\infty$

$= 0.\infty = 1 \neq 3$

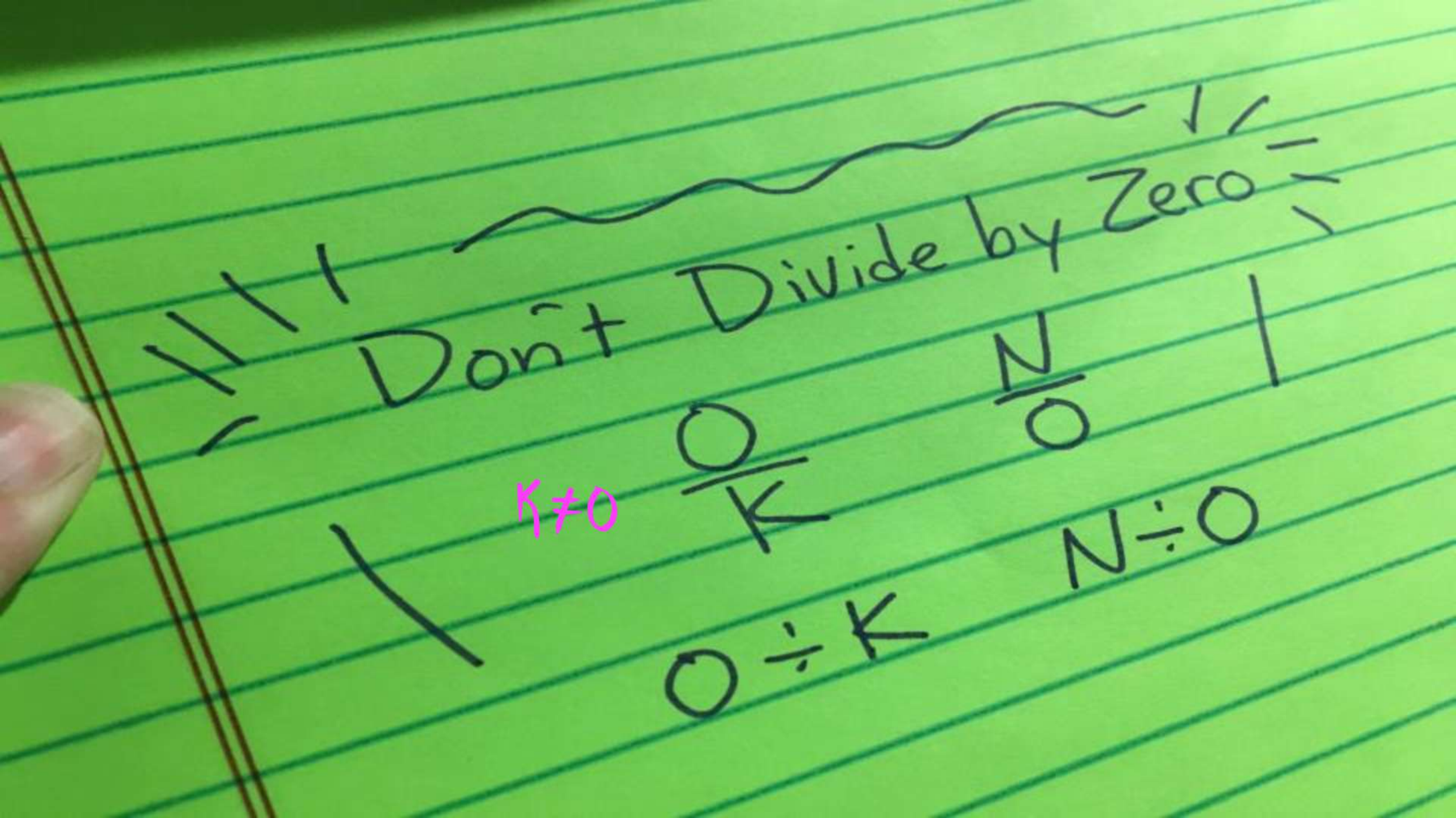
assumption was wrong

ARJUNA



DIVIDE BY ZERO

Congratulations! Your math just destroyed a city.


Don't Divide by Zero
K≠0
0/K
N/0
0÷K
N÷0

BABY, OUR LOVE IS LIKE
DIVIDING BY ZERO
IT CANNOT BE
DEFINED
quickmeme.com

ARJUNA

DIVIDE BY ZERO

Oh shit-

FOR THE LAST TIME

Stop Dividing by Zero

ARJUNA

RESULTS :

(1) : Simplify : $\frac{x-2}{x-2}$

"Maths का Rule"

$D^R \neq 0 \Rightarrow x-2 \neq 0$

$x \neq 2$

$D^R \neq 0$

Ans = 1 ; $x \neq 2$

(2) : Simplify : $\frac{x^2-4}{x-2}$

$D^R \neq 0$

$x-2 \neq 0$

$x \neq 2$

$$\frac{(x-2)(x+2)}{(x-2)} = (x+2);\ x \neq 2$$ Ans.

Q → $\left(\frac{x^2-9}{x+3}\right)$ simplify

$D^R \neq 0$; $x+3 \neq 0$

$x \neq -3$

Ans = $(x-3)$; $x \neq -3$

BABA KA GYAAN →

① $D^R \neq 0$

$\pi - \frac{22}{7} \neq 0$

INFORMATION

π is Rational $\Rightarrow$ "False"

1. $\pi \neq \frac{22}{7}$
2. $\pi \neq 3.14$ $\Rightarrow \pi - 3.14 \neq 0$
3. π is irrational (means its decimal value never ends..)
4. 3.14159265358979323 8..... (NOT LEARN)

ARJUNA

Q. Consider the equation $x^2 - x - \frac{1}{x-1} = 1 - \frac{x}{x-1}$, then x equals - (HW Quesⁿ) ?

1. $\pi - \frac{22}{7} \neq 0$
2. $\pi - 3.14 \neq 0$
3. $\cos 90° = 0$
4. $\cos 0° = 1$

$x - 1 \neq 0$

$\Rightarrow x \neq 1$ ✓

$x^2 - x - \frac{1}{x-1} = \frac{(x-1) - x}{x-1}$ ✓

$x^2 - x = \frac{1}{x-1} - \frac{1}{x-1}$

$x^2 - x = 0$

$x(x-1) = 0 \Rightarrow x = 1, 0$ (1 ↑ Discard) $\Rightarrow x = 0$ ★

ARJUNA

FUFA JI CONCEPT

$\Rightarrow D^R \neq 0$

PROOF OF WAVY CURVE

→ Not Important

(i) $(x-1)(x-2) > 0$

✓ Posi × Posi

✓ Neg × Neg

✗ Posi × Neg

✗ Neg × Posi

Ans → $x \in (-\infty, 1) \cup (2, \infty)$ Ans.

ARJUNA

Q. Solve the linear inequality $1 - \frac{3x}{2} \leq x - 4$ (*HW DISCUSSION*)

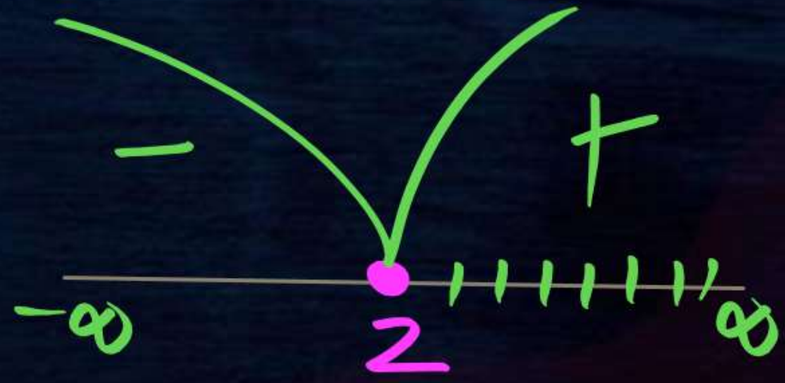
$$1 - \frac{3x}{2} - x + 4 \leq 0$$

$$\left(-\frac{5x}{2} + 5\right) \leq 0$$

$\times(-1)$

$$-\left(-\frac{5x}{2} + 5\right) \geq -0$$

$$\left(\frac{5x}{2} - 5\right) \geq 0$$



$x \in [2, \infty)$ Ans.

$a = -2, 0, 7, 12, \pi, \ldots$

Positive

Negative

zero

"$-a$"

→ zero

ex) $a = 0$

$-a = 0$

★

→ Positive

ex) $a = -2$

$(-a) = 2$

→ Negative

$a = 7$

$-a = -7$

$(a-1)$ → zero $(a=1)$

↳ value depends on 'a'

↳ Positive $(a>1)$

↳ Negative $(a<1)$

$(a+7)$

zero
$a = -7$

Positive
$a = -6$

Negative
$a = -12$

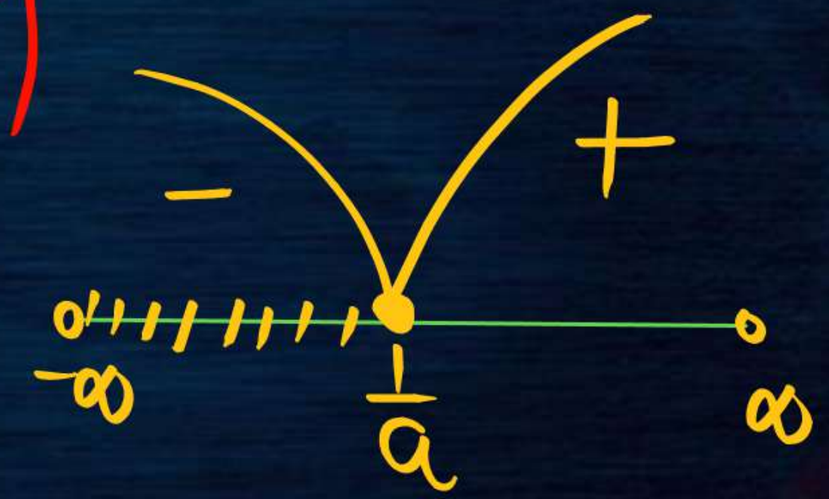
Q. Solve the linear inequality $ax \leq 1$ (HW DISCUSSION)

$(ax-1) \leq 0$

a → Positive
Negative
zero

Case I ⇒ $a > 0$

$(ax-1) \leq 0$



$x \in (-\infty, \frac{1}{a}]$

ARJUNA

Case-2

$a = 0$

$0 \cdot x - 1 \leq 0$

$-1 \leq 0$

Valid

always TRUE

"NEW"

$x \in R$ OR $x \in (-\infty, \infty)$

Case-3

$a<0$ → $-a>0$

$a=-2, (-a)=2$

$ax-1 \leq 0$

a = Negative

$\times(-1)$

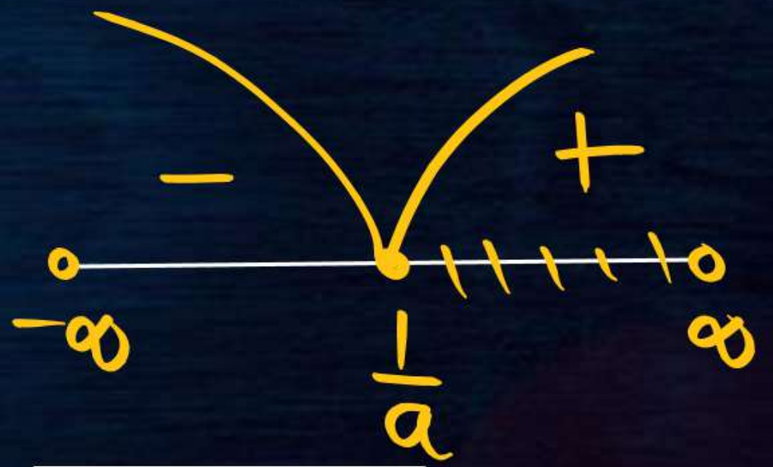
$(-a)x+1 \geq 0$

$-a$ = Positive ✓

$-ax+1=0$

$-ax=-1$

$x=\frac{1}{a}$



$x \in \left[\frac{1}{a}, \infty\right)$

ARJUNA

Final ans →

If $a>0$; $x \in (-\infty, \frac{1}{a}]$

$a=0$, $x \in (-\infty, \infty)$

$a<0$; $x \in [\frac{1}{a}, \infty)$

Summary →

Always TRUE ⇒ $x \in R$ OR $x \in (-\infty, \infty)$

Always False ⇒ $x \in \phi$ OR "No values of x"

$x \in$ No solution.

Q. $a^2 + ax < 1 - x$. Solve for x.

" AKB Sawaal"

" HW Q"

?

SOLVING SYSTEM OF INEQUALITIES (INTERSECTION)

common value nikaalna.

→ Two or more inequalities will be given.

→ Wo 'x' find karo Jo saari inequalities ko satisfy karein.

ARJUNA

Q. Solve the system of inequalities:

✓ $2(3x-1) < 3(4x+1) + 16$, ✓

② $4(2+x) < 3x + 8$ ✓

} Intersecⁿ lo

?

① $6x - 2 < 12x + 3 + 16$

$-6x - 21 < 0$

$6x + 21 > 0$

$x \in \left(-\frac{7}{2}, \infty\right)$ ✓

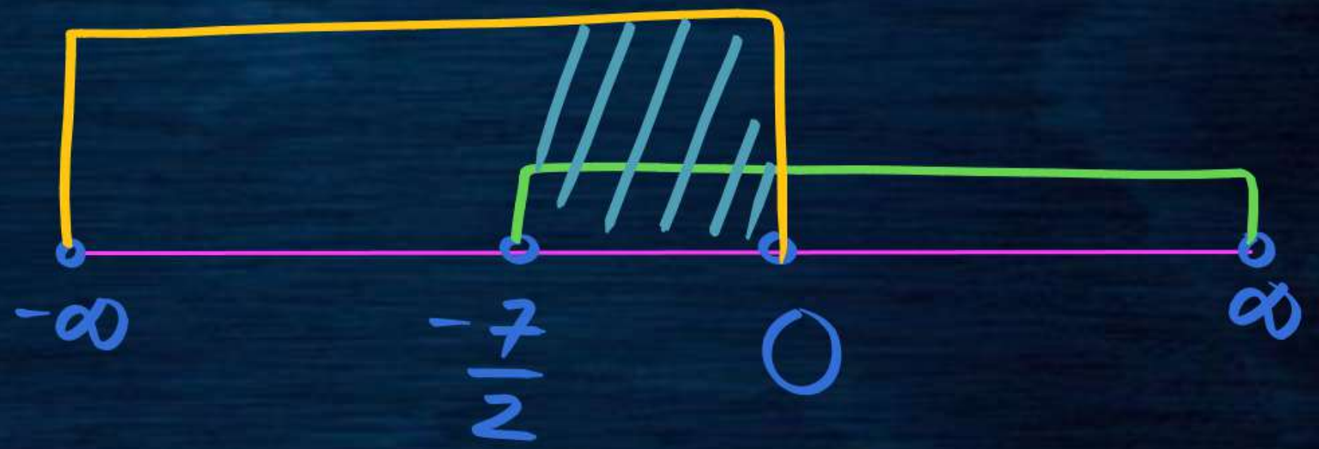
② $8 + 4x < 3x + 8$

$x < 0$

$x \in (-\infty, 0)$ ✓

ARJUNA

Taking Intersec^n →



Common Region →

$\left(-\frac{7}{2}, 0\right)$

Ans.

Q. Solve for 'x' in the following system inequations:-

$$\begin{cases} x^2 + x - 6 < 0 \\ x + 2 \le 0 \\ x^2 - x - 2 > 0 \end{cases}$$

?

$(x^2+x-6) < 0$

$(x+3)(x-2) < 0$

$x \in (-3, 2)$ ✓

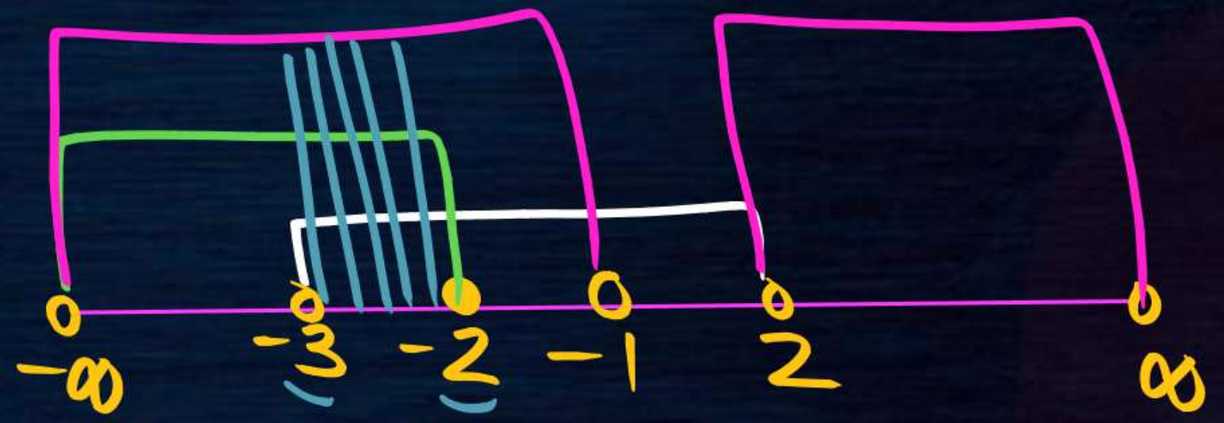
$(x+2) \le 0$

$x \in (-\infty, -2]$ ✓

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$x \in (-\infty, -1) \cup (2, \infty)$ ✓



$x \in (-3, -2]$

Ans.

ARJUNA

Q. Solve: (i) $-11 \leq 4x - 3 \leq 13$

"DOUBLE INEQUALITY"

Method 1

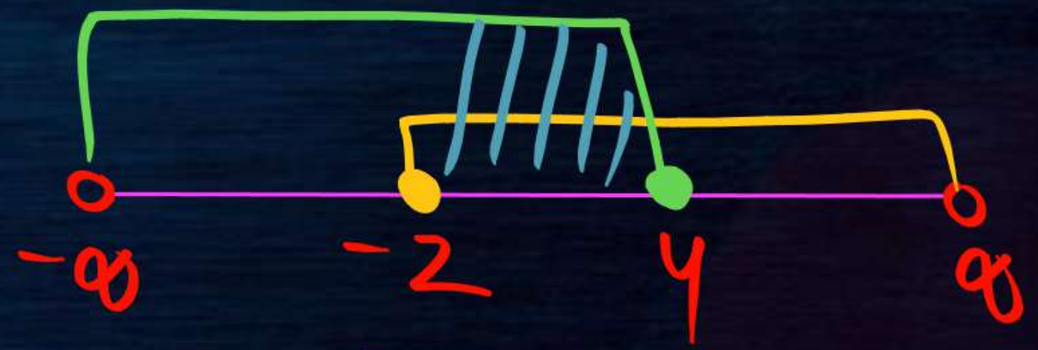
"READ AS"

$$\begin{cases} 4x-3 \geq -11 \\ \text{and} \\ 4x-3 \leq 13 \end{cases}$$

$4x-3 \geq -11 \rightarrow x \in [-2, \infty)$ ✓

$4x-3 \leq 13 \rightarrow x \in (-\infty, 4]$ ✓

Wo 'x' Find Karo, which satisfy Both (Intersecn)



$x \in [-2, 4]$ Ans

"Recommended method"

ARJUNA

Method 2 $\Rightarrow$

$$-11 \le 4x-3 \le 13$$

$+3$

$$-11+3 \le 4x-3+3 \le 13+3$$

$$-8 \le 4x \le 16$$

multiply by "$\frac{1}{4}$"

$$\boxed{-2 \le x \le 4}$$

OR

$$x \in [-2, 4]$$

Ans

Observation No. 3 : *We never cross multiply in any inequality.* ★

$$\frac{x+1}{(x+3)} < 2$$

CROSS MULTIPLY

$$\left(\frac{x+1}{x+3}\right)(x+3) < 2(x+3)$$

WRONG DANGER

$$(x+1) < 2(x+3)$$

(x+3) का sign नहीं पता

"Division wali Inequalities"

RATIONAL INEQUALITIES

Method :

1. Wavy-Baba wala method.
2. Fufa Ji Ka Khayal.
3. Never cross multiply.

ARJUNA

1) $\frac{x+1}{x+3} < 2$ Solve For x.

$x+3 \neq 0$; $x \neq -3$

N.C.M ✓

Har linear factor mai coeff. of x must be positive.

$\frac{x+1}{x+3} - 2 < 0$

$\frac{x+1-2x-6}{x+3} < 0 \Longrightarrow \frac{(-x-5)}{x+3} < 0$

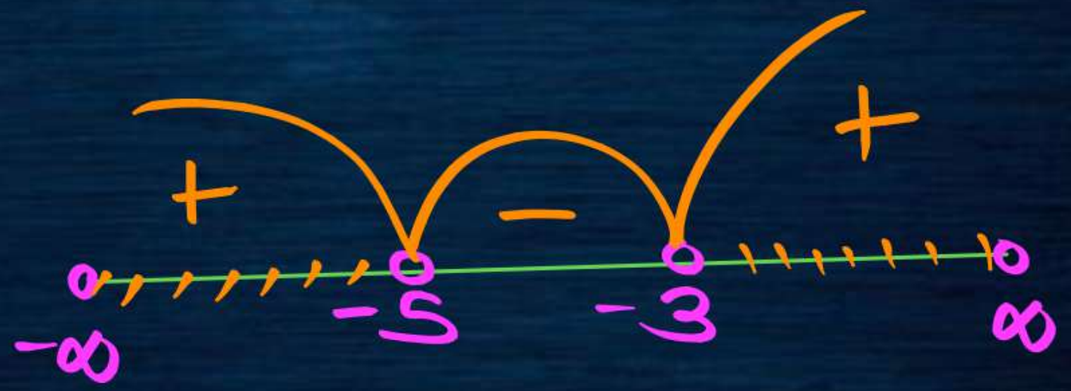
$\times(-1)$

$(-)\ \frac{(-x-5)}{x+3} > 0$

$\frac{x+5}{x+3} > 0$ ✓

ARJUNA

$$\frac{x+5}{(x+3)} > 0$$



$$x \in (-\infty, -5) \cup (-3, \infty)$$

Ans

2) $\frac{2x-7}{x-5} \leq 3$, Solve for x

$x \neq 5$

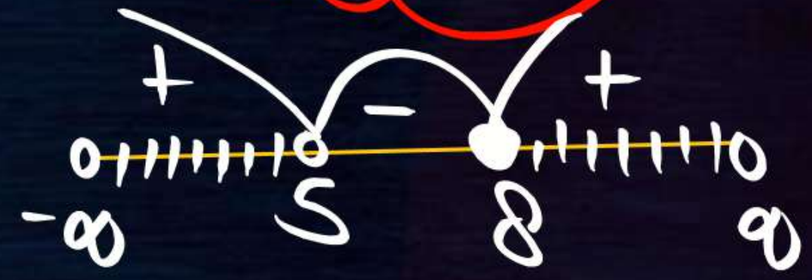
$$\frac{2x-7}{x-5} - 3 \leq 0$$

$$\frac{2x-7-3x+15}{x-5} \leq 0$$

$$\frac{(-x+8)}{x-5} \leq 0$$

$\times(-1)$

$$\frac{x-8}{x-5} \geq 0$$



$x \in (-\infty, 5) \cup [8, \infty)$

Ans

ARJUNA

3) $\frac{x}{x-2} > \frac{1}{x+3}$, Solve for 'x'

N.C.M ; $x \neq 2$, $x \neq -3$

$$\frac{x}{x-2} - \frac{1}{x+3} > 0$$

$$\frac{x(x+3)-(x-2)}{(x-2)(x+3)} > 0$$

$$\frac{x^2+3x-x+2}{(x-2)(x+3)} > 0$$

$$\frac{x^2+2x+2}{(x-2)(x+3)} > 0$$

Challenge

H.W.

DPP1 → complete

DPP2 → Q1, Q2, Q3, Q4, Q5,
Q6, Q7, Q8, Q13,

JBK..JBK...

- TO *FUTURE IIT-IANS*

ARJUNA